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On $(2, k)$-regular and totally $(2, k)$-regular fuzzy graphs

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Abstract

In this paper, we define d_2 -degree and total d_2 -degree of a vertex in fuzzy graphs. Further we study $(2, k)$-regularity and totally $(2, k)$-regularity of fuzzy graphs and the relation between $(2, k)$-regularity and totally $(2, k)$-regularity. Also we study $(2, k)$-regularity on path on four vertices, barbell graph $B_{n,n}, n > 1$ and cycle C_n with some specific membership functions.

Keywords: Regular fuzzy graphs, total degree, totally regular fuzzy graph, d_2 degree of a vertex in graphs, semiregular graphs.
AMS Subject Classification(2010): 05C12, 03E72, 05C72.

1 Introduction

In 1965, Lofti A.Zadeh[12] introduced the concept of a fuzzy subset of a set as a method for representing the phenomena of uncertainty in real life situation. Azriel Rosenfeld introduced fuzzy graphs in 1975[12], which is growing fast and has numerous applications in various fields. Nagoor Gani and Radha [11] introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs. Alison Northup [2] studied some properties on (2,k)-regular graphs in her bachelor thesis. N.R. Santhi Maheswari and C. Sekar introduced d_2 of a vertex in graphs[13] and also discussed some properties on d_2 of a vertex in graphs[14]. Further they introduced $(r, 2, k)$-regular graphs and studied some properties on $(r, 2, k)$-regular graphs[15]. In this paper, we define d_2 -degree and total d_2-degree of a vertex in fuzzy graphs. Further we study $(2, k)$-regularity and totally $(2, k)$-regularity of fuzzy graphs and the relation between $(2, k)$-regularity and totally $(2, k)$-regularity. Also we study $(2, k)$-regularity on path on four vertices, barbell graph $B_{n,n}, n > 1$ and cycle C_n with some specific membership functions.

2 Some Definitions

We give some known definitions as a ready reference for the present study.

Definition 2.1. For a given graph G, the d_2-degree of a vertex v in G, denoted by $d_2(v)$ means number of vertices at a distance two away from v.

Definition 2.2. A graph G is said to be $(2,k)$-regular (d_2-regular) if $d_2(v) = k$, for all v in G. We observe that $(2,k)$-regular graphs and semiregular graphs and d_2-regular graphs are the same.

Definition 2.3. A graph G is said to be $(r,2,k)$-regular if $d(v) = r$ and $d_2(v) = k$, for all v in G.

Definition 2.4. A Fuzzy graph denoted by $G : (\sigma, \mu)$ on graph $G^* : (V, E)$ is a pair of functions (σ, μ) where $\sigma : V \to [0,1]$ is a fuzzy subset of a non empty set V and $\mu : V \times V \to [0,1]$ is a symmetric fuzzy relation on σ such that for all u, v in V the relation $\mu(u,v) = \mu(uv) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph G is complete if $\mu(u,v) = \mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ where uv denotes the edge between u and v. $G^* : (V, E)$ is called the underlying crisp graph of the fuzzy graph $G : (\sigma, \mu)$. σ and μ are called membership function.

Definition 2.5. Let $G : (\sigma, \mu)$ be a fuzzy graph. The degree of a vertex u is $d_G(u) = \sum_{u \neq v} \mu(uv)$ for $uv \in E$ and $\mu(uv) = 0$ for uv not in E; this is equivalent to $d_G(u) = \sum_{uv \in E} \mu(uv)$.

Definition 2.6. The strength of connectedness between two vertices u and v is $\mu^{\infty}(u,v) = sup\{\mu^k(u,v) /k = 1, 2, \dots\}$ where $\mu^k(u,v) = sup\{\mu(uu_1) \wedge \mu(u_1u_2) \wedge \dots \cdots \wedge \mu(u_{k-1}v)/u, u_1, u_2, \dots, u_{k-1}, v$ is a path connecting u and v of length $k\}$.

Definition 2.7. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If $d(v) = k$ for all $v \in V$, then G is said to be regular fuzzy graph of degree k.

Definition 2.8. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The total degree of a vertex u is defined as $td(u) = \sum \mu(u,v) + \sigma(u) = d(u) + \sigma(u)$, $uv \in E$. If each vertex of G has the same total degree k, then G is said to be totally regular fuzzy graph of degree k or k-totally regular fuzzy graph.

Remark 2.9. Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ denote two fuzzy graphs. Let $G_1^* : (V_1, E_1)$ and $G_2^* : (V_2, E_2)$ be respectively the underlying crisp graph such that $|V_i| = p_i, i = 1, 2$. Also $d^*_{Gi}(u_i)$ denotes degree of u_i in G_i^*.

3 d_2-degree of a vertex in fuzzy graphs

Definition 3.1. Let $G : (\sigma, \mu)$ be a fuzzy graph. The d_2-degree of a vertex u in G is $d_2(u) = \sum \mu^2(u,v)$, where $\mu^2(u,v) = sup\{\mu(u,u_1) \wedge \mu(u_1,v)\}$. Also $\mu(uv) = 0$, for uv not in E.
The minimum d_2-degree of G is $\delta_2(G) = \wedge\{d_2(v) : v \in V\}$.
The maximum d_2-degree of G is $\Delta_2(G) = V\{d_2(v) : v \in V\}$.

Example 3.2. Consider $G^* : (V, E)$ where $V = \{u, v, w, x, y\}$ and $E = \{uv, vw, wx, xy, yu\}$. Define $G : (\sigma, \mu)$ by $\sigma(u) = .2, \sigma(v) = .6, \sigma(w) = .5, \sigma(x) = .4, \sigma(y) = .3$ and $\mu(uv) = .1, \mu(vw) = .3, \mu(wx) = .3, \mu(xy) = .2, \mu(yu) = .2$

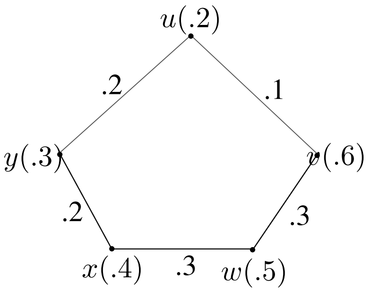

Figure 1

Here, $d_2(u) = \{.1 \wedge .3\} + \{.2 \wedge .2\} = .1 + .2 = .3.$

$d_2(v) = \{.1 \wedge .2\} + \{.3 \wedge .3\} = .1 + .3 = .4.$

$d_2(w) = \{.3 \wedge .2\} + \{.3 \wedge .1\} = .2 + .1 = .3.$

$d_2(x) = \{.2 \wedge .2\} + \{.3 \wedge .3\} = .2 + .3 = .5.$

$d_2(y) = \{.1 \wedge .2\} + \{.2 \wedge .3\} = .1 + .2 = .3.$

Example 3.3. Consider $G^* : (V, E)$ where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx, xu\}$. Define $G : (\sigma, \mu)$ by $\sigma(u) = .2, \sigma(v) = .3, \sigma(w) = .4, \sigma(x) = .5$ and $\mu(uv) = .1, \mu(vw) = .3, \mu(wx) = .4, \mu(xu) = .2$.

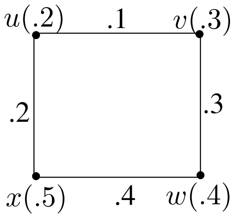

Figure 2

Here, $d_2(u) = Sup\{.1 \wedge .3, .2 \wedge .4\} = Sup\{.1, .2\} = .2,$

$d_2(v) = Sup\{.1 \wedge .2, .3 \wedge .4\} = Sup\{.1, .3\} = .3,$

$d_2(w) = Sup\{.4 \wedge .2, .3 \wedge .1\} = Sup\{.2, .1\} = .2,$

$d_2(x) = Sup\{.2 \wedge .1, .4 \wedge .3\} = Sup\{.1, .3\} = .3.$

4 $(2, k)$-regular and totally $(2, k)$-regular graphs

Definition 4.1. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If $d_2(v) = k$ for all $v \in V$, then G is said to be $(2, k)$-regular fuzzy graph.

Example 4.2. Consider $G^* : (V, E)$ where $V = \{u, v, w, x, y\}$ and $E = \{uv, vw, wx\}$. Define $G : (\sigma, \mu)$ by $\sigma(u) = .2, \sigma(v) = .3, \sigma(w) = .4, \sigma(x) = .5$, and $\mu(uv) = .2, \mu(vw) = .2, \mu(wx) = .2$.

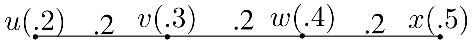

Figure 3

Here $d_2(u) = .2, d_2(v) = .2, d_2(w) = .2, d_2(x) = .2$. This graph is a $(2, .2)$- regular fuzzy graph.

Definition 4.3. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The total d_2-degree of a vertex $u \in V$ is defined as $td_2(u) = \sum \mu^2(u, v) + \sigma(u) = d_2(u) + \sigma(u)$.

Definition 4.4. If each vertex of G has the same total d_2 - degree k, then G is said to be totally $(2, k)$-regular fuzzy graph.

Example 4.5. A totally $(2, k)$-regular fuzzy graph need not be a $(2, k)$-regular fuzzy graph.
Consider $G^* : (V, E)$ where $V = \{u, v, w, x, y\}$ and $E = \{uv, vw, wx, xy, yu\}$.

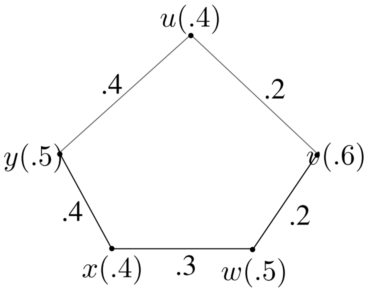

Figure 4

Here $d_2(u) = .6, d_2(v) = .4, d_2(w) = .5, d_2(x) = .6, d_2(y) = .5$ and $td_2(u) = 1, td_2(v) = 1, td_2(w) = 1, td_2(x) = 1, td_2(y) = 1$. Each vertex has same total d_2 -degree 1. So G is totally $(2, 1)$-regular fuzzy graph. But G is not $(2, k)$-regular fuzzy graph.

Example 4.6. A $(2, k)$-regular fuzzy graph need not be a totally $(2, k)$-regular fuzzy graph.
Consider $G^* : (V, E)$ where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx, xu\}$.

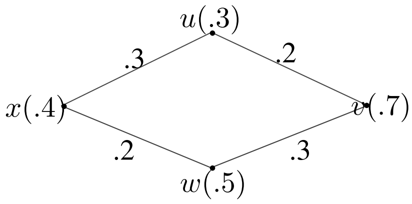

Figure 5

Here $d_2(u) = .2, d_2(v) = .2, d_2(w) = .2, d_2(x) = .2$ and $td_2(u) = .5, td_2(v) = .9, td_2(w) = .7, td_2(x) = .6$. Each vertex has the same d_2-degree .2. So G is $(2, .2)$-regular fuzzy graph. But G is not a totally $(2, k)$-regular fuzzy graph.

Example 4.7. A $(2, k)$-regular fuzzy graph which is totally $(2, k)$-regular fuzzy graph. Consider $G^* : (V, E)$ where $V = \{u, v, w, x, y, z\}$ and $E = \{uv, vw, wx, xy, yz, zu\}$.

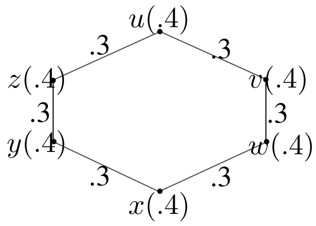

Figure 6

Here $d_2(u) = .6, d_2(v) = .6, d_2(w) = .6, d_2(x) = .6, d_2(y) = .6, d_2(z) = .6$ and $td_2(u) = .1, td_2(v) = 1, td_2(w) = 1, td_2(x) = 1, td_2(y) = 1, td_2(z) = 1$. Each vertex has the same d_2-degree 6. So G is a $(2, .6)$-regular fuzzy graph. Each vertex has the same total d_2-degree 1. So G is a totally $(2, 1)$-regular fuzzy graph.

Theorem 4.8. Let $G : (\sigma, \mu)$ be fuzzy graph on $G^* : (V, E)$. Then $\sigma(u) = c$, for all $u \in V$ if and only if the following conditions are equivalent.

1. $G : (\sigma, \mu)$ is a $(2, k)$-regular fuzzy graph.
2. $G : (\sigma, \mu)$ is a totally $(2, k + c)$-regular fuzzy graph.

Proof: Suppose that $\sigma(u) = c$, for all $u \in V$.
Assume that $G : (\sigma, \mu)$ is a $(2, k)$-regular fuzzy graph. Then $d_2(u) = k$, for all $u \in V$.
Hence, $td_2(u) = d_2(u) + \sigma(u)$, for all $u \in V \Rightarrow td_2(u) = k + c$, for all $u \in V$. Hence, $G : (\sigma, \mu)$ is a totally $(2, k + c)$-regular fuzzy graph.
Thus $(1) \Rightarrow (2)$ is proved.
Suppose $G : (\sigma, \mu)$ is a totally $(2, k + c)$-regular fuzzy graph.

$$\Rightarrow td_2(u) = k + c, \quad for\ all\ u \in V.$$
$$\Rightarrow d_2(u) + \sigma(u) = k + c, \quad for\ all\ u \in V.$$
$$\Rightarrow d_2(u) + c = k + c, \quad for\ all\ u \in V.$$
$$\Rightarrow d_2(u) = k, \quad for\ all\ u \in V.$$

Hence $G : (\sigma, \mu)$ is a $(2, k)$-regular fuzzy graphs. Hence (1) and (2) are equivalent. Conversely assume that (1) and (2) are equivalent. Let $G : (\sigma, \mu)$ is a totally $(2, k+c)$-regular fuzzy graph and $(2, k)$-regular fuzzy graph.

$$\Rightarrow td_2(u) = k + c\ and\ d_2(u) = k, \quad for\ all\ u \in V.$$
$$\Rightarrow d_2(u) + \sigma(u) = k + c\ and\ d_2(u) = k, \quad for\ all\ u \in V.$$
$$\Rightarrow d_2(u) + \sigma(u) = k + c\ and\ d_2(u) = k, \quad for\ all\ u \in V.$$
$$\Rightarrow \sigma(u) = c, \quad for\ all\ u \in V.$$

Hence $\sigma(u) = c$, for all $u \in V$. ∎

5 $(2, k)$-regularity on a path on four vertices with some specific membership functions

In this section, $(2, k)$-regularity on a path on four vertices is studied with some specific membership functions.

Theorem 5.1. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is a path on four vertices. Then, $G : (\sigma, \mu)$ is a $(2, k)$-regular fuzzy graph if $\mu(uv) = k$ for all $uv \in E$.

Proof: Suppose that μ is a constant function say $\mu(uv) = k$ for all $uv \in E$, then $d_2(v) = k$, for all $v \in V$. Hence, G is a $(2, k)$-regular fuzzy graph. ■

Remark 5.2. Converse of Theorem 5.1 need not be true. For example, consider a fuzzy graph $G : (\sigma, \mu)$ such that $G^* : (V, E)$ is a path on four vertices.

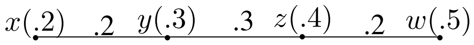

Figure 7

Here $d_2(x) = .2, d_2(y) = .2, d_2(z) = .2, d_2(w) = .2$. So G is $(2, .2)$-regular. But μ is not a constant function.

Theorem 5.3. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is a path on four vertices. If the alternate edges have the same membership values, then G is a $(2, k)$-regular fuzzy graph, where $k = min\{c_1, c_2\}$.

Theorem 5.4. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is a path on four vertices. If middle edge have membership value less than membership value of the remaining edges, then G is a $(2, k)$-regular fuzzy graph, where k =membership value of the middle edge. For example, Consider a fuzzy graph $G : (\sigma, \mu)$ such that $G^* : (V, E)$ is a path on four vertices.

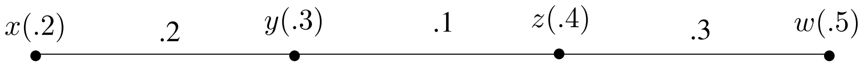

Figure 8

Here $d_2(x) = .1, d_2(y) = .1, d_2(z) = .1, d_2(w) = .1$. So G is $(2, .1)$-regular.

Remark 5.5. If σ is not a constant function, then the $(2, k)$-regular fuzzy graphs in Theorems 5.1, 5.3 and 5.4 are not totally $(2, k)$-regular fuzzy graphs.

6 $(2, k)$-regularity on Barbell graph $B_{n,n}(n > 1)$ with some specific membership functions

In this section, $(2, k)$-regularity on barbell graph $B_{n,n}(n > 1)$. is studied with some specific membership functions

Theorem 6.1. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is a Barbell graph $B_{n,n}$ of order $2n$. If μ is a constant function, then G is a $(2, k)$-regular fuzzy graph where $k = n\mu(uv)$.

Remark 6.2. Converse of Theorem 6.1 need not be true. For example, consider a fuzzy graph $G : (\sigma, \mu)$ such that $G^* : (V, E)$ is a barbell graph $B_{2,2}$ of order 6.

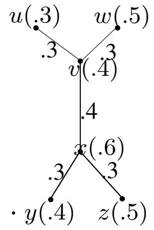

Figure 9

Here, $d_2(u) = .6, d_2(v) = .6, d_2(w) = .6, d_2(x) = .6, d_2(y) = .6$. This graph is a $(2, .6)$-regular fuzzy graph. But μ is not a constant function.

Theorem 6.3. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is the barbell graph $B_{n,n}(n > 1)$. If the pendant edges have the same membership values less than or equal to membership value of the middle edge, then G is a $(2, nk)$-regular fuzzy graph where k = membership value of the pendant edges.

Remark 6.4. If σ is not a constant function, then the $(2, k)$-regular fuzzy graphs in Theorems 6.1 and 6.3 are not totally $(2, k)$-regular fuzzy graphs.

7 $(2, k)$-regularity on a cycle with some specific membership functions

In this section, $(2, k)$-regularity on cycle C_n is studied with some specific membership functions

Theorem 7.1. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is cycle of length ≥ 4. If μ is a constant function, then G is a $(2, k)$-regular fuzzy graph where $k = 2\mu(uv)$.

Remark 7.2. Converse of Theorem 7.1 need not be true. For example, consider a fuzzy graph $G : (\sigma, \mu)$ such that $G^* : (V, E)$ is an odd cycle of length five.

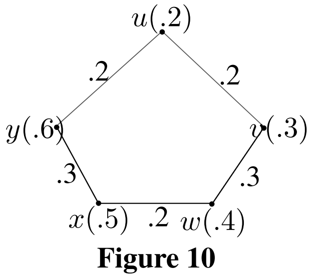

Figure 10

Here, $d_2(u) = .4, d_2(v) = .4, d_2(w) = .4, d_2(x) = .4, d_2(y) = .4$. So G is a $(2, .4)$-regular fuzzy graph. But μ is not a constant function.

Theorem 7.3. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an even cycle. If the alternate edges have the same membership values, then G is a $(2, k)$-regular fuzzy graph.

Proof: If the alternate edges have the same membership values, then

$$\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ c_2, & \text{if } i \text{ is even.} \end{cases}$$

If $c_1 = c_2$, then μ is a constant function. So G is a $(2, 2c_1)$-regular fuzzy graph. If $c_1 < c_2$, then $d_2(v) = 2c_1$, for all $v \in V$. So G is a $(2, 2c_1)$-regular fuzzy graph. If $c_1 > c_2$, then $d_2(v) = 2c_2$, for all $v \in V$. So G is a $(2, 2c_2)$-regular fuzzy graph. ∎

Remark 7.4. Even if the alternate edges of a fuzzy graph whose underlyig graph is an even cycle have the same membership values, G need not be a totally $(2, k)$-regular fuzzy graph, since if σ is not a constant function then G is not a totally $(2, k)$-regular fuzzy graph.

Remark 7.5. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an odd cycle of length ≥ 5. Even if the alternate edges have the same membership values, G need not be $(2, k)$-regular fuzzy graph, since if σ is not a constant function then G is not a totally $(2, k)$-regular fuzzy graphs.
For example, consider a fuzzy graph $G : (\sigma, \mu)$ such that $G^* : (V, E)$ is an odd cycle of length five.

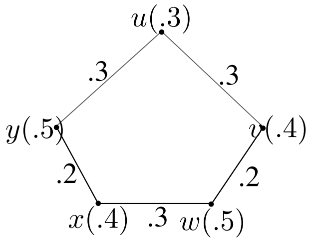

Figure 11

Here $d_2(u) = .4, d_2(v) = .5, d_2(w) = .4, d_2(x) = .4, d_2(y) = .5. d_2(x) \neq d_2(y)$. So G is not a $(2, k)$-regular fuzzy graph.

Theorem 7.6. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is any cycle of length ≥ 4.
Let $\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ c_2 \geq c_1 & \text{if } i \text{ is even.} \end{cases}$
Then G is a $(2, k)$-regular fuzzy graph.

Proof: Let $\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ c_2 \geq c_1, & \text{if } i \text{ is even} \end{cases}$
Case 1. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an even cycle of length ≤ 4. $d_2(v_i) = \{c_1 \wedge c_2\} + \{c_2 \wedge c_1\} = c_1 + c_1 = 2c_1$, for all $v \in V$. So G is a $(2, 2c_1)$-regular fuzzy graph.
Case 2. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an odd cycle of length ≥ 5. Let $e_1, e_2, e_3, \ldots, e_{2n+1}$ be the edges of an odd cycle of G^* in that order.

$$d_2(v_1) = \{\mu(e_1) \wedge \mu(e_2)\} + \{\mu(e_{2n}) \wedge \mu(e_{2n+1})\}$$

$$
\begin{aligned}
&= \{c_1 \wedge c_2\} + \{c_2 \wedge c_1\} = c_1 + c_1 = 2c_1. \\
d_2(v_2) &= \{\mu(e_1) \wedge \mu(e_{2n+1})\} + \{\mu(e_2) \wedge \mu(e_3)\} \\
&= \{c1 \wedge c_1\} + \{c_2 \wedge c_1\} = c_1 + c_1 = 2c_1. \\
&\text{For } i = 3, 4, 5, \ldots, 2n \\
d_2(v_i) &= \{\mu(e_{i-1}) \wedge \mu(e_{i-2})\} + \{\mu(e_{i+1}) \wedge \mu(e_{i+2})\} \\
&= \{c_1 \wedge c_2\} + \{c_2 \wedge c_1\} = c_1 + c_1 = 2c_1. \\
d_2(v_{2n}) &= \{\mu(e_1) \wedge \mu(e_{2n+1})\} + \{\mu(e_{2n}) \wedge \mu(e_{2n-1})\} \\
&= \{c_1 \wedge c_1\} + \{c_2 \wedge c_1\} = c_1 + c_1 = 2c_1. \\
d_2(v_i) &= 2c_1, \text{ for all } v \in V.
\end{aligned}
$$

So G is a $(2, 2c_1)$-regular fuzzy graph. ∎

Remark 7.7. Let $G : (\sigma, \mu)$ be fuzzy graph such that $G^* : (V, E)$ is any cycle of length ≥ 4.
Even if $\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ c_2 \geq c_1, & \text{if } i \text{ is even} \end{cases}$
G need not be a totally $(2, k)$-regular fuzzy graph, since if σ is not a constant function then G is not a totally $(2, k)$-regular fuzzy graph.

Theorem 7.8. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an odd cycle of length ≥ 5 and
$\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ \text{membership value } x \geq c_1 & \text{if } i \text{ is even.} \\ \text{where } x \text{ is not a constant} & \end{cases}$
Then G is a $(2, k)$-regular fuzzy graph.

Proof: Let $\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ \text{membership value } x \geq c_1 & \text{if } i \text{ is even} \\ \text{where } x \text{ is not a constant} & \end{cases}$
Case 1: Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an even cycle of length ≥ 4. $d_2(v_i) = \{c_1 \wedge x\} + \{x \wedge c_1\} = c_1 + c_1 = 2c_1$, for all $v \in V$. So G is a $(2, 2c_1)$-regular fuzzy graph.
Case 2: Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an odd cycle of length ≥ 5. Let $e_1, e_2, e_3, \ldots, e_{2n+1}$ be the edges of an odd cycle of G^* in that order.

$$
\begin{aligned}
d_2(v_1) &= \{\mu(e_1) \wedge \mu(e_2)\} + \{\mu(e_{2n}) \wedge \mu(e_{2n+1})\} \\
&= \{c_1 \wedge x\} + \{x \wedge c_1\} = c_1 + c_1 = 2c_1. \\
d_2(v_2) &= \{\mu(e_1) \wedge \mu(e_{2n+1})\} + \{\mu(e_2) \wedge \mu(e_3)\} \\
&= \{c_1 \wedge c_1\} + \{x \wedge c_1\} = c_1 + c_1 = 2c_1.
\end{aligned}
$$

For $i = 3, 4, 5, \ldots, 2n$

$$\begin{aligned} d_2(v_i) &= \{\mu(e_{i-1}) \wedge \mu(e_{i-2})\} + \{\mu(e_{i+1}) \wedge \mu(e_{i+2})\} \\ &= \{c_1 \wedge x\} + \{x \wedge c_1\} = c_1 + c_1 = 2c_1. \\ d_2(v_{2n}) &= \{\mu(e_1) \wedge \mu(e_{2n+1})\} + \{\mu(e_{2n}) \wedge \mu(e_{2n-1})\} \\ &= \{c_1 \wedge c_1\} + \{x \wedge c_1\} = c_1 + c_1 = 2c_1. \\ d_2(v_i) &= 2c_1,\ for\ all\ v \in V. \end{aligned}$$

So G is a $(2, 2c_1)$-regular fuzzy graph. ∎

Remark 7.9. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an odd cycle of length ≥ 5.

Even if $\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ membership\ value\ x \geq c_1 & \text{if } i \text{ is even,} \\ where\ x\ is\ not\ a\ constant & \end{cases}$

then G need not be a totally $(2, k)$-regular fuzzy graph, since if σ is not a constant function then G is not a totally $(2, k)$-regular fuzzy graph.

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