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Prime cordial labeling of some special graph families

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Abstract

A bijection f from vertex set V of a graph G to $\{1, 2, \ldots, |V|\}$ is called a prime cordial labeling of G if each edge uv is assigned the label 1 if $gcd(f(u), f(v)) = 1$ and 0 if $gcd(f(u), f(v)) > 1$, where the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. In this paper we exhibit some new constructions on prime cordial graphs.

Keywords: Petersen graph, fan, flower, cycle with triangle, prime cordial graph.
AMS Subject Classification(2010): 05C78.

1 Introduction

Graph labeling is a strong relation between numbers and structure of graphs. A useful survey to know about the numerous graph labeling methods is given by J. A. Gallian[5]. By combining the relatively prime concept in number theory and cordial labeling concept[4] in graph labeling, Sundaram et al.[10] introduced the concept called prime cordial labeling. A bijection f from vertex set $V(G)$ to $\{1, 2, \ldots, |V(G)|\}$ of a graph G is called a prime cordial labeling of G if for each edge $e = uv \in E$,

$$\begin{aligned} f^*(e = uv) &= 1; \text{ if } gcd(f(u), f(v)) = 1 \\ &= 0; \text{ if } gcd(f(u), f(v)) > 1 \end{aligned}$$

then $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ is the number of edges labeled with 0 and $e_f(1)$ is the number of edges labeled with 1.

In [1],[2],[9], the following graphs are proved to have prime cordial labeling: C_n if and only if $n \geq 6$; P_n if and only if $n \neq 3$ or 5; $K_{1,n}(n\ odd)$; the graph obtained by subdividing each edge of $K_{1,n}$ if and only if $n \geq 3$.

S. K. Vaidya et al.[11],[12],[13] proved that the square graph of path P_n is a prime cordial graph for $n = 6$ and $n \geq 8$ while the square graph of cycle C_n is a prime cordial graph for $n \geq 10$. They also proved that the shadow graph of $K_{1,n}$ for $n \geq 4$, the shadow graph of $B_{n,n}$ for all n, certain cycle related graphs, the graphs obtained by mutual duplication of a pair of edges as well as mutual duplication of a pair of vertices from each of two copies of cycle C_n admit prime cordial labeling. Haque et al.[7] proved that the generalized Petersen graph is prime cordial. S. Babitha and J. Baskar Babujee[3] exhibit some characterization results and new constructions on prime cordial graphs. G. V. Ghodasara and J. P. Jena[6] discussed prime cordial labeling for the graph related to cycle with one chord, cycle with twin chord and cycle with triangle.

Definition 1.1. The Petersen graph is $3-$regular undirected graph with 10 vertices and 15 edges.

Definition 1.2. The fan graph is denoted by F_n and described as $F_n = P_n + K_1$, where P_n indicates the path graph with n vertices.

Definition 1.3. The helm H_n is the graph obtained from a wheel graph W_n by attaching a pendant vertex through an edge to each rim vertex of W_n.

Definition 1.4. The flower Fl_n is the graph obtained from a helm H_n by joining each pendant vertex of the helm to the apex vertex. Here the pendant vertices of helm H_n are referred as external vertices of Fl_n.

Definition 1.5. A cycle with triangle is a cycle with three chords which by themselves form a triangle.

For positive integers p, q, r and $n \geq 6$ with $p + q + r + 3 = n$, $C_n(p, q, r)$ denotes a cycle with triangle whose edges form the edges of cycles C_{p+2}, C_{q+2} and C_{r+2} without chords.

Notation 1.6. The floor and ceiling functions map a real number to the largest previous or the smallest following integer respectively. More precisely, $floor(x) = \lfloor x \rfloor$ is the largest integer not greater than x and $ceiling(x) = \lceil x \rceil$ is the smallest integer not less than x.

2 Main Results

Theorem 2.1. The graph G obtained by joining two copies of Petersen graph by a path of arbitrary length is prime cordial.

Proof: Let G be the graph obtained by joining two copies of Petersen graph by a path P_k of length $k - 1$. Let $u_1, u_2, \dots u_5$ and $u_6, u_7, \dots u_{10}$ be external and internal vertices of first copy of petersen graph respectively. Here each u_i is adjacent to u_{i+5}, $i = 1, 2, 3, 4, 5$. Similarly let $w_1, w_2, \dots w_5$ and $w_6, w_7, \dots w_{10}$ be external and internal vertices of second copy of petersen graph respectively. Here each w_i is adjacent to w_{i+5}, $si = 1, 2, 3, 4, 5$. Let $v_1, v_2, \dots v_k$ be successive vertices of path P_k with $v_1 = u_1$ and $v_k = w_1$.

We define a labeling function $f : V(G) \to \{1, 2, \dots, k + 18\}$ as follows.

$f(u_i) = 4i - 3; 1 \leq i \leq 5,$

$\quad = 4(i - 5) - 1; 6 \leq i \leq 10,$

$f(v_j) = 2i + 17; 2 \leq j \leq \lceil \frac{k}{2} \rceil,$

$= 2i - \lceil \frac{k}{2} \rceil + 20; (\lceil \frac{k}{2} \rceil + 1) \leq i \leq k-1,$

$f(w_i) = 4i - 2; 1 \leq i \leq 5,$

$= 4(i-5); 6 \leq i \leq 10.$

The labeling defined above satisfies the conditions of prime cordial labeling and hence the graph under consideration is a prime cordial graph. ■

Illustration 2.2. A prime cordial labeling of the graph obtained by joining two copies of the Petersen graph by a path P_8 is shown in Figure 1.

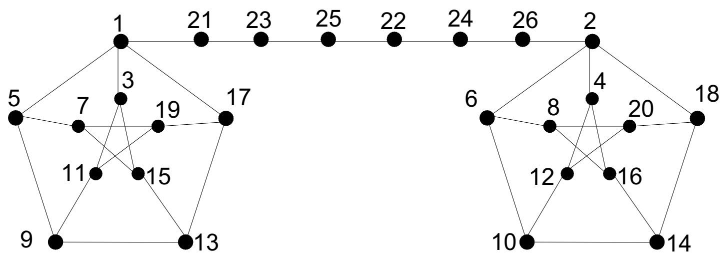

Figure 1: Prime cordial labeling of graph obtained by joining two copies of petersen graph by a path P_8.

Theorem 2.3. The graph G obtained by joining two copies of fan graph F_n by a path of arbitrary length is prime cordial.

Proof: Let G be the graph obtained by joining two copies of fan graph F_n by a path P_k of length $k-1$. Let us denote the successive vertices of first copy of fan graph by $u_1, u_2, \ldots, u_{n+1}$ and the successive vertices of second copy of fan graph by $w_1, w_2, \ldots, w_{n+1}$. Let $v_1, v_2, \ldots, v_k$ be the vertices of path P_k with $v_1 = u_1$ and $v_k = w_1$. Note that for $n = 2$, F_2 is a cycle C_3 and it is already proved in [11] that the graph obtained by joining two copies of cycles by a path of arbitrary length is prime cordial. Hence we consider the case for $n \geq 3$. We define a labeling function $f : V(G) \rightarrow \{1, 2, \ldots, 2n+k-2\}$ as follows.

Case 1: k is even.

In this case define f as:

$f(u_1) = f(v_1) = 2$, $f(w_1) = f(v_k) = 1$,

$f(u_2) = 4$, $f(v_{\frac{k}{2}}) = 6$,

$f(u_i) = k + 2(i-1); 3 \leq i \leq n,$

$f(v_j) = 6 + 2(j-1); 2 \leq j \leq \frac{k}{2} - 1$

$= 2j - k + 1; \frac{k}{2} + 1 \leq j \leq k-1,$

$f(w_i) = k + 2i - 3; 1 \leq i \leq n.$

Case 2: k is odd.

In this case define f as:

$f(u_1) = f(v_1) = 2$, $f(w_1) = f(v_k) = 1$,

$f(u_2) = 4$, $f(v_{\frac{k-1}{2}}) = 6$,

$f(u_i) = k + 2i - 3; 3 \leq i \leq n$,

$$f(v_j) = 6 + 2(j-1); 2 \leq j \leq \tfrac{k-3}{2}$$
$$= 2(j+1) - k; \tfrac{k+1}{2} \leq j \leq k-1,$$
$$f(w_i) = k + 2(i-1); 1 \leq i \leq n.$$

In each case f satisfies the conditions of prime cordial labeling and hence the graph under consideration is a prime cordial graph. ∎

Illustration 2.4. Prime cordial labeling of the graph obtained by joining two copies of F_8 by a path P_7 is shown in Figure 2.

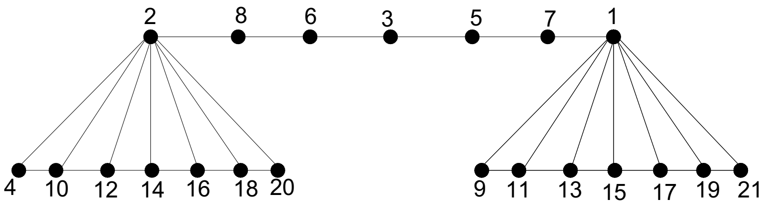

Figure 2: Prime cordial labeling of graph obtained by joining two copies of F_8 by path P_7.

Theorem 2.5. The graph G obtained by joining two copies of flower graph Fl_n by a path of arbitrary length is prime cordial.

Proof: Let G be the graph obtained by joining two copies of flower graph Fl_n by a path P_k of length $k-1$. Let u_0 be the apex vertex, $u_1, u_2, \ldots, u_n$ be the rim vertices and $u'_1, u'_2, \ldots, u'_n$ be the external vertices of first copy of flower Fl_n. Let w_0 be the apex vertex, $w_1, w_2, \ldots, w_n$ be the rim vertices and $w'_1, w'_2, \ldots, w'_n$ be the external vertices of second copy of flower Fl_n. Let $v_1, v_2, \ldots, v_k$ be the vertices of path P_k with $v_1 = u_1$ and $v_k = w_1$.

We define a labeling function $f : V(G) \to \{1, 2, \ldots, 2n + k - 2\}$ as follows.

Case 1: $k = 2$.

In this case define f as:

$f(u_1) = f(v_1) = 4$,

$f(w_1) = f(v_2) = 3$,

$f(u_0) = 2$, $f(w_0) = 1$,

$f(w_1') = 7,$

$f(u_i) = 2(i+1); 1 \leq i \leq n,$

$f(u_i') = 2(n+i+1); 1 \leq i \leq n,$

$$\begin{aligned} f(w_i) &= 8i - 1; 2 \leq i \leq \lceil \tfrac{n}{2} \rceil \\ &= 8(n-i) + 9; (\lceil \tfrac{n}{2} \rceil + 1) \leq i \leq n, \end{aligned}$$

$$\begin{aligned} f(w_i') &= 8i - 5; 2 \leq i \leq \lceil \tfrac{n}{2} \rceil \\ &= 8(n-i) + 5; (\lceil \tfrac{n}{2} \rceil + 1) \leq i \leq n. \end{aligned}$$

Case 2: $k = 3$.

In this case define f as:

$f(u_1) = f(v_1) = 4,$

$f(v_2) = 4n + 3,$

$f(w_1) = f(v_3) = 3,$

$f(u_0) = 2, f(w_0) = 1,$

$f(w_1') = 7,$

$f(u_i) = 2(i+1); 1 \leq i \leq n,$

$f(u_i') = 2(n+i+1); 1 \leq i \leq n,$

$$\begin{aligned} f(w_i) &= 8i - 1; 2 \leq i \leq \lceil \tfrac{n}{2} \rceil \\ &= 8(n-i) + 9; (\lceil \tfrac{n}{2} \rceil + 1) \leq i \leq n, \end{aligned}$$

$$\begin{aligned} f(w_i') &= 8i - 5; 2 \leq i \leq \lceil \tfrac{n}{2} \rceil \\ &= 8(n-i) + 5; (\lceil \tfrac{n}{2} \rceil + 1) \leq i \leq n. \end{aligned}$$

Case 3: $k \geq 3$.

In this case define f as:

$f(u_0) = 2, f(w_0) = 1,$

$f(w_1) = f(v_k) = 3,$

$f(w_1') = 7,$

$f(v_{\lfloor \frac{k}{2} \rfloor}) = 4,$

$f(u_i) = 2(i+2); 1 \leq i \leq n,$

$f(u_i') = 2(n+i+2); 1 \leq i \leq n,$

$$\begin{aligned} f(w_i) &= 8i - 1; 2 \leq i \leq \lceil \tfrac{n}{2} \rceil \\ &= 8(n-i) + 9; (\lceil \tfrac{n}{2} \rceil + 1) \leq i \leq n, \end{aligned}$$

$$\begin{aligned} f(w_i') &= 8i - 5; 2 \leq i \leq \lceil \tfrac{n}{2} \rceil \\ &= 8(n-i) + 5; (\lceil \tfrac{n}{2} \rceil + 1) \leq i \leq n, \end{aligned}$$

$f(v_j) = 4n + 2(j + 1); 2 \leq j \leq \lfloor \frac{k}{2} \rfloor - 1$
$\quad = 4n + 2(j - \lfloor \frac{k}{2} \rfloor) + 1; \lceil \frac{k}{2} \rceil \leq j \leq k.$

One can observe that in each case the labeling defined above satisfies the conditions of prime cordial labeling and the graph under consideration is a prime cordial graph. ■

Illustration 2.6. The prime cordial labeling of the graph obtained by joining two copies of Fl_6 by s path P_9 is shown in Figure 3.

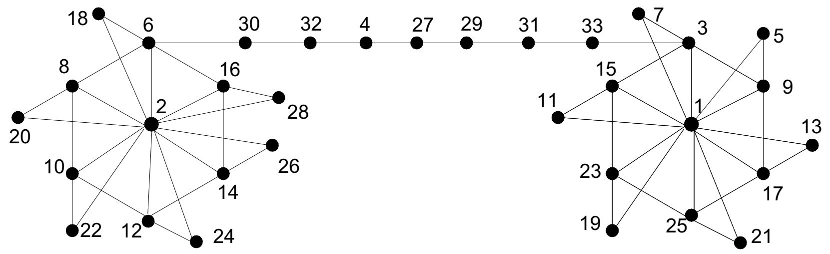

Figure 3: Prime cordial labeling of the graph obtained by joining two copies of Fl_6 by a path P_9.

Theorem 2.7. The graph G obtained by joining two copies of cycle C_n with triangle by a path of arbitrary length is prime cordial.

Proof: Let G be the graph obtained by joining two copies of cycle C_n with triangle by path P_k of length $k - 1$. Let $u_1, u_2, \ldots, u_n$ be the vertices of first copy of cycle with triangle. Let $w_1, w_2, \ldots, w_n$ be the vertices of second copy of cycle with triangle. Let $v_1, v_2, \ldots, v_k$ be the vertices of path P_k with $u_1 = v_1$ and $v_k = w_1$. Let $e_1 = u_1u_3$, $e_2 = u_3u_5$, $e_3 = u_5u_1$ be the chords in first copy of cycle C_n and $e'_1 = w_1w_3$, $e'_2 = w_3w_5$, $e'_3 = w_5w_1$ be the chords in second copy of cycle C_n. We define a labeling function $f : V(G) \to \{1, 2, \ldots, 2n + k - 2\}$ as follows.

Case 1: k is even.

In this case define f as:

$f(u_1) = f(v_1) = 1,$
$f(w_1) = f(v_k) = k,$
$f(u_2) = 3, f(u_3) = 9, f(u_4) = 5, f(u_5) = 7,$
$f(u_i) = 2i - 1; 6 \leq i \leq n,$
$f(v_j) = 2n + 2j - 3; 2 \leq i \leq \frac{k}{2}$
$\quad = 2j - k; \frac{k}{2} + 1 \leq i \leq k,$
$f(w_i) = k + 2(i - 1); 1 \leq i \leq n.$

Case 2: k is odd.

In this case define f as:

$f(u_1) = f(v_1) = 1,$
$f(w_1) = f(v_k) = k - 1,$

$f(u_2) = 3, f(u_3) = 9, f(u_4) = 5, f(u_5) = 7,$
$f(u_i) = 2i - 1; 6 \leq i \leq n,$
$f(v_j) = 2n + 2j - 3; 2 \leq i \leq \frac{k+1}{2}$
$\quad = 2j - (k+1); \frac{k+3}{2} \leq i \leq k,$
$f(w_i) = k + 2i - 3; 1 \leq i \leq n.$

In each case, the labeling defined above satisfies the conditions of prime cordial labeling and hence the graph under consideration is a prime cordial graph. ■

Illustration 2.8. The prime cordial labeling of the graph obtained by joining two copies of C_7 with triangle by a path P_6 is shown in Figure 4. It is the case related to k is even.

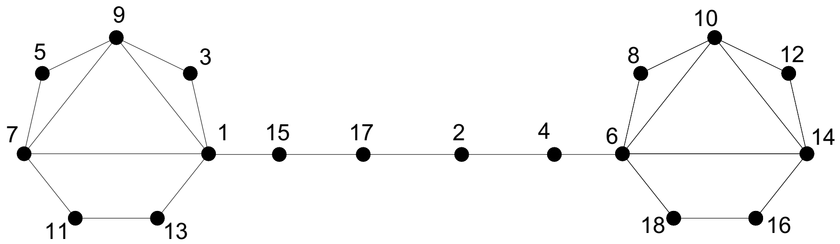

Figure 4: Prime cordial labeling of graph obtained by joining two copies of C_7 with triangle by a path P_6.

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